MODELS FOR DYNAMIC CONDITIONS IN GRANULATION

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Models are derived for the dynamic conditions in granulation in an apparatus in which the suspension is fed into the jet.

There are ongoing increases in granulator unit power and tighter specifications for product quality, which requires research on the dynamic conditions. There is an entire range of equipments, usually of low power, which work in the quasistationary state and in which the product tapoff and/or the supply of recirculated material are performed at fixed time instants. It is impossible to optimize such an equipment without a reasonably exact model for the dynamic conditions. This explains the considerable attention [1-3] given to the description and study of dynamic granule formation. However, all existing models consider the granulator as a system homogeneous in material balance (an object with lumped parameters).

That approach is applicable for certain types of granulator but is completely unacceptable for describing the processes in current equipments (Fig. 1) in which the suspension is fed into the active jet [4, 5]. In fact, the processes in the jet differ considerably from those in the rest of the volume, and the assumption that there is ideal mixing of the particles throughout the volume results in substantial errors.

Here we propose models for the dynamic conditions of granulation in an apparatus in which the suspension is fed into the jet and the recirculated material is fed also into the jet and/or into the bulk of the apparatus, on the basis that there are substantial differences in the conditions in the jet and in the rest of the volume. The models provide for continuous and cyclic recirculation and tapoff.

To construct the model we consider the apparatus as a system consisting of two interacting zones: the jet and the bulk (Fig. 2), where the bulk zone is taken as the entire volume of the apparatus excluding the jet. The figure shows that particles with a grain-size composition of the bulk pass from the bulk into the jet zone, while ones with a grain-size composition of the jet pass from the jet into the bulk. Part of the recirculated material Q_p' enters the jet and another part Q_p'' enters the bulk (note that Q_p' or Q_p'' may be absent).

Making the usual assumptions [1], we consider that the bulk zone is a one-pool object with lumped parameters, i.e., we assume that the particles in the bulk are ideally mixed. This assumption, as in [1], is justified by the vigorous circulation. As in [1-4], we assume that the particles are spherical.

We also make the following additional assumptions: 1) the probability of a particle entering the jet is independent of the size, and 2) the number of particles leaving the jet at time t is equal to the number of particles entering the jet at time $t - \bar{\tau}_j$.

By $\bar{\tau}$ we denote the mean time between successive entries of a particle into the jet. According to the assumption of [1], this time is the same for all the particles. It consists of the mean time $\bar{\tau}_j$ spent in the jet and the mean time $\tau - \tau_j$ spent outside the jet.

By N we denote the total number of particles in the apparatus, and get

$$N = n_{in}\bar{\tau}. \tag{1}$$

The mean time spent in the jet is defined as

$$\bar{\tau}_j = \frac{N_{in}}{n_{in}}. \tag{2}$$

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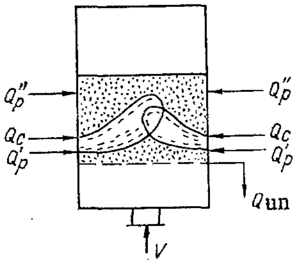

Fig. 1

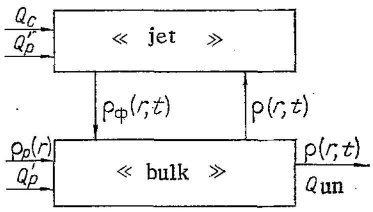

Fig. 2

Fig. 1. Granulator apparatus.

Fig. 2. Structural scheme for apparatus.

It follows from (1) and (2) that

$$N_{\mathrm{j}} = \frac{\bar{\tau}_{\mathrm{in}}}{\bar{\tau}} N. \tag{3}$$

We thus have a model for the processes in the jet. We assume that all of the suspension is deposited on the particles in the jet in proportion to their surfaces:

$$Q_c(t) = 4\pi N_{\mathrm{j}} \int_0^\infty \lambda r^2 \rho_{\mathrm{j}}(r,\ t)\, dr, \tag{4}$$

where λ is a coefficient of proportionality constant at each instant (the rate of deposition of the suspension on unit surface). Then the change in volume V of a particle can be found from

$$\frac{dV}{dt} = \lambda S, \tag{5}$$

whence

$$\frac{dr}{dt} = \lambda. \tag{6}$$

The number of particles N_r^{r+dr} of sizes in the range [r, r + dr] is defined (Fig. 3) as follows:

$$N_r^{r+dr} = N_{\mathrm{j}} \rho_{\mathrm{j}}(r,\ t)\, dr.$$

The change ΔN_r^{r+dr} in N_r^{r+dr} in the time interval [t, t + dt] is put as

$$\Delta N_r^{r+dr} = \frac{\partial [N_{\mathrm{j}} \rho_{\mathrm{j}}(r,\ t)]}{\partial t} dr dt. \tag{7}$$

We write the overall balance equation for the number of particles in the jet:

$$\Delta N_r^{r+dr} = N_1 - N_2 + N_{\mathrm{in}}^r - N_{\mathrm{out}}^r + \alpha N_p^r, \tag{8}$$

where N_1 is the number of particles whose size has increased during time dt and has entered the range [r, r + dr], N_2 is the number of particles leaving the size range [r, r + dr] in time dt, N_{in}^r is the number of particles of size [r, r + dr] entering from the bulk zone in time dt, N_{out}^r is the number of particles of size [r, r + dr] leaving the jet in time dt, and N_p^r is the proportion of particles of size [r, r + dr] entering the apparatus with the recirculating material.

Clearly, $0 \leqslant \alpha \leqslant 1$, and $\alpha = 0$ corresponds to supplying the recirculating material to the bulk of the apparatus, while $\alpha = 1$ corresponds to supplying the recirculating material completely to the jet.

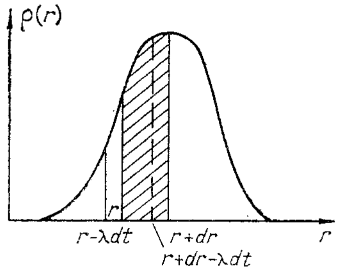

Fig. 3. Probability density distribution for particle radii.

The number of particles N_1 is equal to the number of particles with sizes in the range $[r - \lambda(r, t)dt, r]$ (Fig. 3) and is defined by

$$N_1 = N_j \rho_j(r, t) \lambda(r, t) dt. \tag{9}$$

The number N_2 is equal to the number of particles with sizes in the range $[r + dr - \lambda(r + dr, t)dt, r + dr]$:

$$N_2 = N_j \rho_j (r + dr, t) \lambda(r + dr, t) dt. \tag{10}$$

The components N^r_{in} and N^r_{out} are defined correspondingly by

$$N^r_{in} = n_{in}\rho(r, t) drdt, \quad N^r_{out} = n_{out} \rho_j (r, t) drdt,$$
$$n_{out}(t) = n_{in}[t - \bar{\tau}_j] + \alpha \cdot n_p [t - \bar{\tau}_j], \tag{11}$$

where $n_{in}[t - \bar{\tau}_j]$, $n_p[t - \bar{\tau}_j]$ are the values of n_{in} and n_p displaced by $\bar{\tau}_j$ correspondingly.

The value of N^r_p is defined by

$$N^r_p = n_p \rho_p(r) drdt. \tag{12}$$

We substitute (9)-(12) into (8) and use the fact that $\rho_j(0, \cdot) = \rho_j(\infty, \cdot) = 0$ to get

$$\frac{\partial (N_j \rho_j (r, t))}{\partial t} + N_{out} \frac{\partial (\rho_j (r, t) \lambda (r, t))}{\partial r} = n_j \rho(r, t) - n_{out} \rho_j (r, t) + \alpha n_p \rho_p (r, t) \tag{13}$$

or on the basis that according to the definition $\lambda(r, t)$ is independent of r

$$N_j \frac{\partial \rho_j (r, t)}{\partial t} + N_j \lambda(t) \frac{\partial \rho_j (r, t)}{\partial r} = -\rho_j (r, t) \frac{\partial N_j}{\partial t} + n_{in} \rho(r, t) - n_{out} \rho_j (r, t) + \alpha n_p \rho_p (r, t). \tag{14}$$

On the basis of $\frac{dN_j}{dt} = n_{in} - n_{out} + \alpha n_p$ (14) becomes

$$\frac{\partial \rho_j (r, t)}{\partial t} + \lambda (t) \frac{\partial \rho_j (r, t)}{\partial r} = \frac{n_{in}}{N_j} [\rho (r, t) - \rho_j (r, t)] + \alpha \frac{n_p}{N_j} [\rho_p (r, t) - \rho_j (r, t)]. \tag{15}$$

A model for the bulk zone can be derived similarly.

The change ΔN^{r+dr}_r in the number of particles with sizes in the range $[r, r + dr]$ in the bulk is expressed by

$$\Delta N^{r+dr}_r = \frac{\partial [(N - N_j) \rho (r, t)]}{\partial t} drdt. \tag{16}$$

The following form is given for the balance equation for the number of particles in the range $[r, r + dr]$:

$$\Delta N^{r+dr}_r = N^r_{out} - N^r_{in} - N^r_{un} + (1 - \alpha) N^r_p, \tag{17}$$

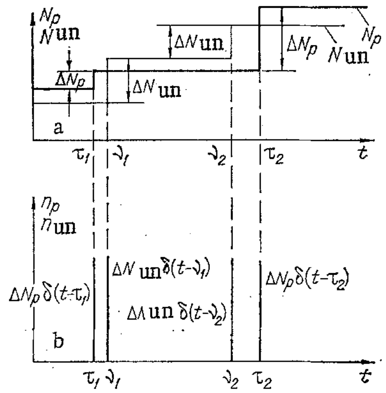

Fig. 4. Integral recycled-material input and finished-product removal (a) and rate of recycled-material input and rate of finished-product removal (b).

where N^r_{un} is the number of particles of the product with sizes in the range [r, r + dr] unloaded in time dt. We determine the value of N^r_{un}:

$$N^r_{un} = n_{un}\rho(r,\ t)\,dr\,dt. \tag{18}$$

We substitute (11), (18), and (12) into (17) to get

$$\frac{\partial[(N-N_j)\rho(r,\ t)]}{\partial t} = n_{out}\,\rho_\Phi(r,\ t) - n_{in}\rho(r,\ t) + (1-\alpha)\,n_p\rho_p(r,\ t) - n_{un}\rho(r,\ t). \tag{19}$$

Therefore, we can describe the change in grain-size composition in a dynamic process in which there is continuous supply of recirculated material and unloading of the product from an apparatus containing a jet on the basis of the following model:

$$Q_c = 4\pi N_j\int_0^\infty \lambda r^2\rho_j(r,\ t)\,dr,$$

$$\frac{\partial\rho_j(r,\ t)}{\partial t} + \lambda\frac{\partial\rho_j(r,\ t)}{\partial r} = \frac{n_{in}}{N_j}[\rho(r,\ t)-\rho_j(r,\ t)] - $$

$$-\alpha\frac{n_p}{N_j}[\rho_p(r,\ t)-\rho_j(r,\ t)],$$

$$\frac{\partial\rho(r,\ t)}{\partial t} = \frac{n_{out}}{N-N_j}[\rho_j(r,\ t)-\rho(r,\ t)] +$$

$$+(1-\alpha)\frac{n_p}{N-N_j}[\rho_p(r,\ t)-\rho(r,\ t)],$$

$$\frac{dN}{dt} = n_p - n_{un},\quad \frac{dN_j}{dt} = n_j - n_{out} + \alpha n_p,\quad n_{in} = N/\bar{\tau},$$

$$Q_p = n_p\frac{4}{3}\pi\int_0^\infty r^3\rho_p(r,\ t)\,dr,\quad Q_{un} = n_{un}\frac{4}{3}\pi\int_0^\infty r^3\rho(r,\ t)\,dr,$$

$$N(0) = N_0,\ \rho_j(\cdot,\ 0) = \rho(\cdot,\ 0) = \rho_0(\cdot),$$

where $\rho_0(r)$ is the initial particle distribution density.

We now give a description of the process when the unloading of the particles and/or the loading with the recirculated material are performed during short intervals at fixed instants. We assume that at the instants τ_i (i = 1, 2, ...) there is instantaneous recycled material input, while at the instants ν_i (i = 1, 2, ...) there is unloading of part of the product. In the general case, the instants τ_i and ν_i may not coincide.

$$n_p = \sum_{i=1}^{\infty} \Delta N_p(t)\,\delta(t-\tau_i), \tag{20}$$

$$n_{\text{un}} = \sum_{i=1}^{\infty} \Delta N_{\text{un}}(t)\,\delta(t-\nu_i), \tag{21}$$

where $\delta(t)$ is a Dirac function.

We use (20) and (21) in (12) and (18) to get a model for the dynamic conditions on periodic supply of the recirculated material and unloading:

$$Q_c = 4\pi N_j \int_0^{\infty} \lambda(t)\, r^2 \rho_j(r, t)\, dr,$$

$$\frac{\partial \rho_j(r, t)}{\partial t} + \lambda \frac{\partial \rho_j(r, t)}{\partial r} = \frac{n_{\text{in}}}{N_j}[\rho(r, t) - \rho_j(r, t)] -$$

$$-\alpha \frac{1}{N_j}\Big[\sum_{i=1}^{\infty} \rho_p(r, t)\,\Delta N_p(t)\,\delta(t-\tau_i) - \sum_{i=1}^{\infty} \rho_j(r, t)\,\Delta N_p(t)\,\delta(t-\tau_i)\Big],$$

$$\frac{\partial \rho(r, t)}{\partial t} = \frac{n_{\text{out}}}{N - N_j}[\rho_j(r, t) - \rho(r, t)]$$

$$+(1-\alpha)\frac{1}{N - N_j}\Big[\sum_{i=1}^{\infty} \rho_p(r, t)\,\Delta N_p(t)\,\delta(t-\tau_i) - \sum_{i=1}^{\infty} \rho(r, t)\,\Delta N_p(t)\,\delta(t-\tau_i)\Big],$$

$$\frac{dN(t)}{dt} = \sum_{i=1}^{\infty} (\Delta N_p(t)\,\delta(t-\tau_i) - \Delta N_{\text{un}}(t)\,\delta(t-\nu_i)),$$

$$\frac{dN_j}{dt} = y(t) - n_{\text{un}}(t),$$

$$y(t) = \alpha \sum_{i=1}^{\infty} \Delta N_p \delta(t-\tau_i) + n_{\text{in}}(t),$$

$$n_{\text{out}}(t) = y[t - \bar{\tau}_j],\quad n_{\text{in}} = N/\tau_j,$$

$$\Delta Q_p = \Delta N_p \frac{4}{3}\pi \int_0^{\infty} r^3 \rho_p(r, t)\, dr,$$

$$\Delta Q_{\text{un}} = \Delta N_{\text{un}} \frac{4}{3}\pi \int_0^{\infty} r^3 \rho(r, t)\, dr,$$

$$N(0) = N_0,\ \rho_j(r, 0) = \rho(r, 0) = \rho_0(r).$$

These models can be used to calculate the time course of the size distribution density and also to control the process. However, it is sometimes sufficient if the designer knows the time course of the mean particle radius. Such a model is also much simpler than a model for the change in the particle distribution density and can be used in many engineering calculations during design and also in granulator management.

We multiply (15) by r and take the integral from 0 to ∞ to get

$$\frac{\partial}{\partial t}\int_0^{\infty} \rho_j(r, t)\, r\,dr + \lambda(t)\int_0^{\infty} \frac{\partial \rho_j(r, t)}{\partial r}\, r\,dr = \frac{n_{\text{in}}}{N_j}\int_0^{\infty} [\rho(r, t)$$

$$-\rho_j(r, t)]\, r\,dr + \alpha\frac{n_p}{N_j}\int_0^{\infty} [\rho_p(r, t) - \rho_j(r, t)]\, r\,dr \tag{22}$$

or after simple transformations

$$\frac{d}{dt}\bar{r}_j = \lambda + \frac{n_{in}}{N_j}(\bar{r}-\bar{r}_j) + \alpha\frac{n_p}{N_j}[\bar{r}_p-\bar{r}_j], \tag{23}$$

where $\bar{r}_j$ is the mean particle radius in the jet, $\bar{r}$ is the mean particle radius in the apparatus, and $\bar{r}_p$ is the mean particle radius in the recirculated material.

These are defined by the following formulas:

$$\bar{r}_j = \int_0^\infty r\rho_j(r,\ t)\,dr,\ \bar{r} = \int_0^\infty r\rho(r,\ t)\,dr,\ \bar{r}_p = \int_0^\infty r_p\rho_p(r,\ t)dr.$$

We make analogous transformations in (19) to get

$$\frac{\partial\bar{r}}{dt} = \frac{n_{out}}{N-N_j}(\bar{r}_j-\bar{r}) + (1-\alpha)\frac{n_p}{N-N_j}(\bar{r}_p-\bar{r}). \tag{24}$$

We multiply (15) by $4\pi r^2$ and take the integral from 0 to ∞ to get

$$\frac{\partial\bar{S}_j}{\partial t} = 8\pi\bar{r}_j\lambda(t) + \frac{n_{in}}{N_j}[\bar{S}-\bar{S}_j] + \alpha\frac{n_p}{N_j}[\bar{S}_p-\bar{S}_j], \tag{25}$$

where $\bar{S}_j$, $\bar{S}$, $\bar{S}_p$ are the mean surface areas of the particles in the jet, the bed, and the recirculated material, which are defined respectively by

$$\bar{S}_j = \int_0^\infty 4\pi r^2\rho_j(r)\,dr,\ \bar{S} = \int_0^\infty 4\pi r^2\rho(r)\,dr,\ \bar{S}_p = \int_0^\infty 4\pi r^2\rho_p(r)\,dr.$$

By analogy with (25) we get from (19) that

$$\frac{d\bar{S}}{dt} = \frac{n_{out}}{N-N_j}[\bar{S}_j-\bar{S}] + (1-\alpha)\frac{n_p}{N-N_j}(\bar{S}_p-\bar{S}). \tag{26}$$

We then write (4) as $Q_c = N_j\lambda\bar{S}_j$.

Then the following is the system of equations describing the change in the mean granule radius in the continuous process:

$$Q_c = N_j\lambda\bar{S}_j,\ \frac{d\bar{r}_j}{dt} = \lambda + \frac{n_{in}}{N_j}[\bar{r}-\bar{r}_j] + \alpha\frac{n_p}{N_j}(\bar{r}_p-\bar{r}_j),$$

$$\frac{dN}{dt} = n_p - n_{un},$$

$$\frac{d\bar{S}_j}{dt} = 8\pi\bar{r}_j\lambda(t) + \frac{n_{in}}{N_j}(\bar{S}-\bar{S}_j) + \alpha\frac{n_p}{N_j}(\bar{S}_p-\bar{S}_j),$$

$$\frac{d\bar{S}}{dt} = \frac{n_{out}}{N-N_j}[\bar{S}_j-\bar{S}] + (1-\alpha)\frac{n_p}{N-N_j}(\bar{S}_p-\bar{S}_j),$$

$$\frac{dN_j}{dt} = n_{in} - n_{out} + \alpha n_p,\ n_{in} = \frac{N}{\tau},$$

$$Q_p = N_p\int_0^\infty \frac{4}{3}\pi r^3\rho_p(r,\ t)\,dr,\ Q_{un} = N_{un}\int_0^\infty \frac{4}{3}\pi r^3\rho(r,\ t)\,dr,$$

$$\bar{r}_j(0) = \bar{r}(0) = \int_0^\infty r\rho_0(r)\,dr,$$

$$\bar{S}_j(0) = \bar{S}(0) = \int_0^\infty 4\pi r^2\rho_0(r)\,dr,\ N(0) = N_0.$$

Similarly, for cyclic supply of the recirculated material and unloading we get

$$Q_c = N_j \lambda \bar{S}_j,$$

$$\frac{d\bar{r}_j}{dt} = \lambda + \frac{n_{in}}{N_j}[\bar{r} - \bar{r}_j] + \frac{\alpha}{N_j}\sum_{i=1}^{\infty}\Delta N_p(t)(\bar{r}_p - \bar{r}_j)\,\delta(t-\tau_i),$$

$$\frac{d\bar{r}}{dt} = \frac{n_{out}}{(N-N_j)}[\bar{r}_j - \bar{r}] + (1-\alpha)\frac{1}{(N-N_j)}\sum_{i=1}^{\infty}\Delta N_p(t)\,\bar{r}_p(t)\,\delta(t-\tau_i) - (1-\alpha)\frac{1}{N-N_j}\sum_{i=1}^{\infty}\Delta N_p(t)\,\bar{r}(t)\,\delta(t-\tau_i),$$

$$\frac{dN}{dt} = \sum_{i=1}^{\infty}\Delta N_p(t)\,\delta(t-\tau_i) - \sum_{i=1}^{\infty}\Delta N_{un}(t)\,\delta(t-\nu_i),$$

$$\frac{d\bar{S}_j}{dt} = 8\pi\bar{r}_j\lambda(t) + \frac{n_{in}}{N_j}[\bar{S} - \bar{S}_j] + \frac{\alpha}{N_j}\sum_{i=1}^{\infty}\Delta N_p(t)(\bar{S}_p - \bar{S}_j)\,\delta(t-\tau_i),$$

$$\frac{d\bar{S}}{dt} = \frac{n_{un}}{(N-N_j)}[\bar{S}_j - \bar{S}] + (1-\alpha)\frac{1}{N-N_j}\sum_{i=1}^{\infty}\Delta N_p(t)\,\bar{S}_p(t)\,\delta(t-\tau_i)$$

$$-(1-\alpha)\frac{1}{N-N_j}\sum_{i=1}^{\infty}\Delta N_p(t)\,\bar{S}(t)\,\delta(t-\tau_i),$$

$$\frac{dN_j}{dt} = y(t) - n_{out}(t),\quad y(t) = \alpha\sum_{i=1}^{\infty}\Delta N_p\delta(t-\tau_i) + n_{in}(t),$$

$$n_{out}(t) = y[t - \bar{\tau}_j],\quad n_{in} = \frac{N}{\bar{\tau}},$$

$$Q_p = N_p\frac{4}{3}\pi\int_0^{\infty} r^3\rho_p(r,\ t)\,dr,$$

$$Q_{un} = N_{un}\frac{4}{3}\pi\int_0^{\infty} r^3\rho(r,\ t)\,dr,$$

$$\bar{r}_j(0) = \bar{r}(0) = \int_0^{\infty} r\rho_0(r)\,dr,$$

$$\bar{S}_j(0) = \bar{\rho}(0) = \int_0^{\infty} 4\pi r^2\rho_0(r)\,dr,\quad N(0) = N_0.$$

These models enable one to determine the change in grain-size composition in accordance with the point of supply, the flow rate, and the grain-size composition of the recirculated material, the programs for varying the number of particles in the apparatus, and the suspension flow rate. The models can be used to determine the instants τ_i and ν_i for tapping off the product and supplying the recirculated material and also for determining the amount of product tapped off to provide a quasistatic process and a satisfactory product quality.

Apart from the point of recycled material input, the models use parameters such as the time spent by a particle in the jet τ_j and the time between successive entries of a particle to the jet, which are dependent not only on the conditions but also on the design parameters.

Therefore, if appropriate constraints are imposed on the hydrodynamic circumstances in the apparatus and on the qualitative parameters of the product, these models can be used to optimize the working conditions and design parameters.

NOTATION

N, total number of particles in the apparatus; N_j, number of particles in the jet; n_{in}, number of particles entering the jet from the bulk in unit time; n_{out}, number of particles leaving the jet in unit time; $\bar{\tau}$, mean time between successive entrances of a particle into the jet; $\bar{\tau}_j$, mean dwell time in the jet; λ, suspension deposition rate per unit surface; Q'_p, part of the recycled material entering the jet; Q_p'', part of the recycled material entering the bulk; Q_c, suspension flow rate; ρ_j, ρ, distribution densities in the jet and bulk, respectively; r, particle radius; S, particle surface; V, particle volume; t, time; α, fraction of recycled material supplied to the jet; n_{un}, product unloading rate; n_p, recycled-material supply rate; τ_i, instant i of the instantaneous recycled material loading; ν_j, instant j of instantaneous product unloading; Q_p, Q_{un}, flow rates of the recycled material and the final product; N_0, initial number of particles in the apparatus; ΔQ_p, ΔQ_{un}, volumes of the recycled material and product supplied to and withdrawn from the apparatus at a fixed instant.

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IMPROVEMENT OF THE LIMITING RELATIVE FRICTION LAW ON A PERMEABLE PLATE WITH BLOWN GAS

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UDC 532.526:536.24

The authors have improved the correlation for the effect of finite Reynolds number on the critical parameters of blowing and the limiting relative friction law in the incompressible turbulent boundary layer with blowing of gas at the wall.

Asymptotic turbulent boundary-layer theory was used in [1] to obtain the following formulas for the limiting relative friction law and the critical blowing parameters:

for $\psi < 1$

$$\Psi_\infty = \frac{4}{b_1(1-\psi)}\left[\ln\frac{\sqrt{(1-\psi)(1+b_1)}+\sqrt{b_1}}{\sqrt{1-\psi}+\sqrt{b_1\psi}}\right]^2, \tag{1}$$

$$b_{\mathrm{cr}\,\infty} = \frac{1}{1-\psi_1}\left(\ln\frac{1+\sqrt{1-\psi_1}}{1-\sqrt{1-\psi_1}}\right)^2; \tag{2}$$

for $\psi > 1$

$$\Psi_\infty = \frac{4}{b_1(\psi-1)}\left[\operatorname{arctg}\sqrt{\frac{b_1}{(\psi-1)(b_1+1)}} - \operatorname{arctg}\sqrt{\frac{b_1\psi}{\psi-1}}\right]^2, \tag{3}$$

$$b_{\mathrm{cr}\,\infty} = \frac{1}{\psi_1-1}\left(\arccos\frac{2-\psi_1}{\psi_1}\right)^2; \tag{4}$$

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